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Singular system analysis for temperature retrieval in microwave thermography

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This paper refers to the inversion of microwave radiometric data for retrieving the temperature distribution in the human body. The singular system of the radiative integral operator furnishes an appropriate basis, provided some a priori information is included. The relevant features of the technique are discussed and numerical simulations of the retrievals are reported.

1. INTRODUCTION

Temperature distributions in the human body and their variations with time may be characteristic of specific pathologies and indicative of the course of an illness. The measure of the skin temperature gives information on the occurring physiological and pathological processes. Infrared thermography is an effective means of mapping skin surface temperatures, based on the measurement of electromagnetic radiation at infrared wavelengths [*Gautherie and Albert*, 1983]. On the other side, noninvasive monitoring of subcutaneous temperature distributions still remains as an important goal in biomedicine. This kind of measurement would allow not only the direct detection of deep thermal anomalies, but also the control of temperature during the heating of neoplastic tissues in loco-regional hyperthermia treatment of tumors. Microwave radiometers are able to detect the radiation emitted by the inner tissues up to a depth of few centimeters. Because of its potential, microwave radiometry has been proposed and experimented to obtain information on the subcutaneous temperature distribution in the human body. Laboratory and clinical experimental validation of microwave radiometry has been conducted by using single-frequency systems. Processing of data has yielded significant results, particularly when referred to the detection of left-right asymmetries in breast cancer cases [*Edrich*, 1979; *Myers et al.*, 1979].

Indications on the temporal thermal variations produced by hyperthermic treatments have been obtained also [*Chive' et al.*, 1984]. Multispectral radiometry has been recently considered as a technique able to provide information about human inner temperature distributions [*Miyakawa*, 1981; *Bardati and Solimini*, 1983], analogously to what is being done in atmospheric sounding [*Westwater*, 1972]. Dual-frequency and three-band systems have been tested on models and animals to assess the visibility of thermal structures [*Bocquet et al.*, 1986] and to identify particular temperature profiles [*Mizushina et al.*, 1986].

In this paper we are concerned with the use of the multispectral radiometric technique for thermal mapping in the human body. The core of the technique is the retrieval of the temperature pattern from values of the thermal emission measured at diverse microwave frequencies. Three-layer structures consisting of skin-fat-muscle and muscle-bone-muscle arrangements are considered, since they are representative of the essential local emitting properties of several regions of the body. The emission from a one-

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dimensional structure is given in terms of a brightness temperature T_B:

$$T_B(\omega) = \int_{-D}^{0} W(\omega, z)T(z)\, dz \qquad (1)$$

where $\omega = 2\pi f$ is the angular frequency of a monochromatic radiometric channel, D is a penetration depth, i.e., the depth within the tissues beyond which the contribution to the emission is negligible, and $T(z)$ is the temperature profile to be retrieved. The kernel $W(\omega, z)$ of (1) is called "weighting function" and its general expression has been given by *Bardati and Solimini* [1984].

The inverse problem of retrieving the temperature from brightness measurements can be modeled as the solution of a first-kind Fredholm integral equation. This is a typical example of an ill-posed problem: the solution is not stable with respect to small variations of the data. This difficulty can be circumvented if one recognizes that only a finite amount of information about the solution can be extracted from the data, in the sense that only a finite number of components of the solution with respect to a suitable basis can be accurately determined [*Twomey*, 1974; *Pike et al.*, 1984]. The appropriate basis is provided by the singular system of the integral operator involved in the equation to be solved. A difficulty inherent to the approach is that the singular functions of the problem are discontinuous across the interfaces between different tissues. As a consequence, the retrieved thermodynamic temperature does not have the correct continuity properties. To avoid this difficulty, a suitable space of functions (Sobolev space) is introduced and continuous singular functions are considered. The relevant aspects of the retrieval technique are then summarized, referring the reader interested in mathematical details to another paper [*Bardati et al.*, 1987]. In the numerical simulation discussed in the following section 3, we assume that the brightness temperature is known for a sufficiently large number of frequencies in the band 1.5–6.5 GHz. The amount of information which can be extracted from the data is quantified both in the case of the SFM and in the case of the MBM structure. Examples of retrievals of pulse functions are reported for the two structures and the effect of varying the thickness of one layer is considered. The reconstructions can be interpreted in terms of the impulse response of the system, consisting of the radiometer plus the algorithm, hence they give indication on the resolution limits of the method.

2. INVERSION METHOD

Equation (1) defines an integral operator $\mathscr{L}$ from space X of square-integrable functions $T(z)$ into space Y of square-integrable functions $T_B(\omega)$, for $\omega_1 \le \omega \le \omega_2$. X and Y are equipped with norms

$$\| T(z) \|_X = \left\{ (1/D) \int_{-D}^{0} T^2(z)\, dz \right\}^{1/2} \qquad (2)$$

$$\| T_B(\omega) \|_Y = \left\{ [1/(\omega_2 - \omega_1)] \int_{\omega_1}^{\omega_2} T_B^2(\omega)\, d\omega \right\}^{1/2} \qquad (3)$$

Since $W(\omega, z)$ is bounded and z, ω can range only over bounded intervals, operator $\mathscr{L}$ is compact and hence admits a singular system $\{\alpha_j;\ u_j(z);\ v_j(\omega)\}_{j=1}^{\infty}$ [*Miller*, 1974], which is the set of the solutions of the coupled equations

$$\mathscr{L}_{u_j} = \alpha_j v_j \qquad \mathscr{L}^* v_j = \alpha_j u_j \qquad (4)$$

where $\mathscr{L}^*$ is the adjoint operator

$$(\mathscr{L}^* T_B)(z) = [D/(\omega_2 - \omega_1)] \int_{\omega_1}^{\omega_2} W(\omega, z) T_B(\omega)\, d\omega \qquad (5)$$

$$-D \le z \le 0$$

Singular functions $u_j(z)$ form a basis in X while $v_j(\omega)$ form a basis in Y. In order to solve (1), $T(z)$ and $T_B(\omega)$ are expanded as series of $u_j(z)$ and of $v_j(\omega)$, respectively. In absence of noise, the component of $T_B(\omega)$ with respect to $v_j(\omega)$ is given by α_j times the corresponding component of $T(z)$ with respect to $u_j(z)$. In presence of noise, however, the noise contribution to the data with sufficiently large j is larger than the signal contribution. As a consequence, for a given signal-to-noise ratio only a finite number of components of $T(z)$ can be extracted from the data (number of "degrees of freedom"). In practice, only a discrete set of measurements is available. However, if the number N of data is sufficiently large, then the problem with discrete data is a good approximation of the problem with continuous data and the number of degrees of freedom is the same in the two cases.

Let $W_n(z) = W(\omega_n, z)$ be the weighting function corresponding to the nth experimental point, and let $g_n = T_B(\omega_n)$. The following norm

$$\| \mathbf{g} \|_{Y_N} = \left\{ [1/(N-1)] \sum_{n=1}^{N} g_n^2 \right\}^{1/2} \qquad (6)$$

is introduced in the space Y_N of data. Analogously to the case of continuous data, the singular system

$$\{\alpha_{N,j};\ u_{N,j},\ v_{N,j}\}_{j=1}^{\infty} \qquad (7)$$

can be computed for the operator $\mathscr{L}_N$ which maps $T(z)$ from X to Y_N. The singular values $\alpha_{N,j}$ are the square roots of the eigenvalues of the Gram matrix

$$G_{nm} = [D/(N-1)] \int_{-D}^{0} W_n(z) W_m(z)\, dz \tag{8}$$

The singular functions are discontinuous as a consequence of the discontinuities of the weighting functions $W_n(z)$ due to the discontinuous nature of the tissues. It follows that the temperature profile restored on such a basis does not satisfy physical requirements such as the continuity of both $T(z)$ and $K(z) \cdot T'(z)$ at the interface between adjacent media, having different thermal conductivities $K(z)$. To overcome the difficulty, a new space X of continuous solutions of (1) is considered with a scalar product defined by

$$(T, \Phi)_{\bar{X}} = T(0)\Phi(0) + T(-D)\Phi(-D) + (1/H) \cdot \int_{-D}^{0} B(z)T(z)\Phi(z)\, dz + (1/H)\int_{-D}^{0} K(z)T'(z)\Phi'(z)\, dz \tag{9}$$

where $B(z)$ and the constant H are specified on the basis of physical considerations. Therefore we look for functions $\Phi_n(z)$ such that, for any $T(z)$,

$$(T, \Phi_n)_{\bar{X}} = \int_{-D}^{0} T(z) W_n(z)\, dz \tag{10}$$

It can be recognized that function $\Phi_n(z)$ has the correct regularity properties, since it is the temperature distribution in the case of steady state linear heat flow in a one-dimensional structure layered along the z axis, when heat is supplied at the rate $HW_n(z)$ per unit volume.

In the case of discrete data, the problem (1) takes the form

$$(T, \Phi_n)_{\bar{X}} = g_n \qquad n = 1, \cdots, N \tag{11}$$

The solution of (11) is not unique, since any temperature distribution which is orthogonal to the subspace spanned by $\Phi_n(z)$ produces a zero data vector. However, there exists a unique solution of minimal norm (normal solution) in such a subspace. The normal solution can be expressed as

$$T^+(z) = \sum_{j=1}^{N} (1/\alpha_{N,j})(\mathbf{g}, \mathbf{v}_{N,j})_{Y_N} u_{N,j}(z) \tag{12}$$

An important question is the numerical stability of such a solution. As is known [e.g., *Pike et al.*, 1984; *Bertero et al.*, 1985], if $\delta\mathbf{g}$ denotes a "small" variation of the data vector $\mathbf{g}$ and $\delta T^+(z)$ denotes the corresponding variation of the normal solution, the following inequality holds true

$$\frac{\| \delta T^+ \|_{\bar{X}}}{\| T^+ \|_{\bar{X}}} = \frac{\alpha_{N,1}}{\alpha_{N,N}} \cdot \frac{\| \delta \mathbf{g} \|_{Y_N}}{\| \mathbf{g} \|_{Y_N}} \tag{13}$$

The quantity $\alpha_{N,1}/\alpha_{N,N}$ is called the condition number and is a large number as a consequence of the fact that (11) is a discrete version of an ill-posed problem [*Bardati et al.*, 1987]. The condition number is an estimate of the error magnification introduced by the algorithm. Indeed, because of the amplification of the noise corrupting the input data, the normal solution (12) is not useful in practice. Rather, a "regularized," or suitably filtered solution is needed. In terms of singular function expansions, a rather general definition of the filtered solution is the following:

$$\tilde{T}(z) = \sum_{j=1}^{N} (\xi_j/\alpha_{N,j})(\mathbf{g}, \mathbf{v}_{N,j})_{Y_N} u_{N,j}(z) \tag{14}$$

where the ξ_j are suitable window coefficients. The simplest numerical filtering technique consists in using in (14) a number of terms smaller than N (rectangular window), i.e.,

$$\xi_j = 1 \qquad 1 \le j \le M \qquad \xi_j = 0 \qquad M < j \le N \tag{15}$$

Other possibilities are the use of a triangular window

$$\begin{aligned} \xi_j &= 1 - (j-1)/M \qquad 1 \le j \le M \\ \xi_j &= 0 \qquad M < j \le N \end{aligned} \tag{16}$$

of the Hamming window, of the Tikhonov regularizer, etc. [*Bertero et al.*, 1985]. In all these cases the value of a free parameter, like the number M of terms in the windows (15) and (16), must be established. A criterion for the choice of the parameter can be derived through a compromise between numerical stability of the solution and compatibility of the solution with the data. Equation (13) indicates that a choice based on the condition number $\alpha_{N,1}/\alpha_{N,N}$ is able to stabilize the numerical solution against noise in the data. The number M of terms is assumed as large as possible, consistently with $\| \delta T^+ \|_{\bar{X}} = \| T^+ \|_{\bar{X}}$ for a given noise-to-signal ratio $\| \delta \mathbf{g} \|_{Y_N} / \| \mathbf{g} \|_{Y_N}$. Such a criterion is indeed rather pessimistic, in the sense that M can be assumed slightly larger than the value corresponding to the condition number, without an appreciable deterioration of the signal-to-noise ratio in the solution. It has been shown [*Bardati et al.*, 1987] that the following

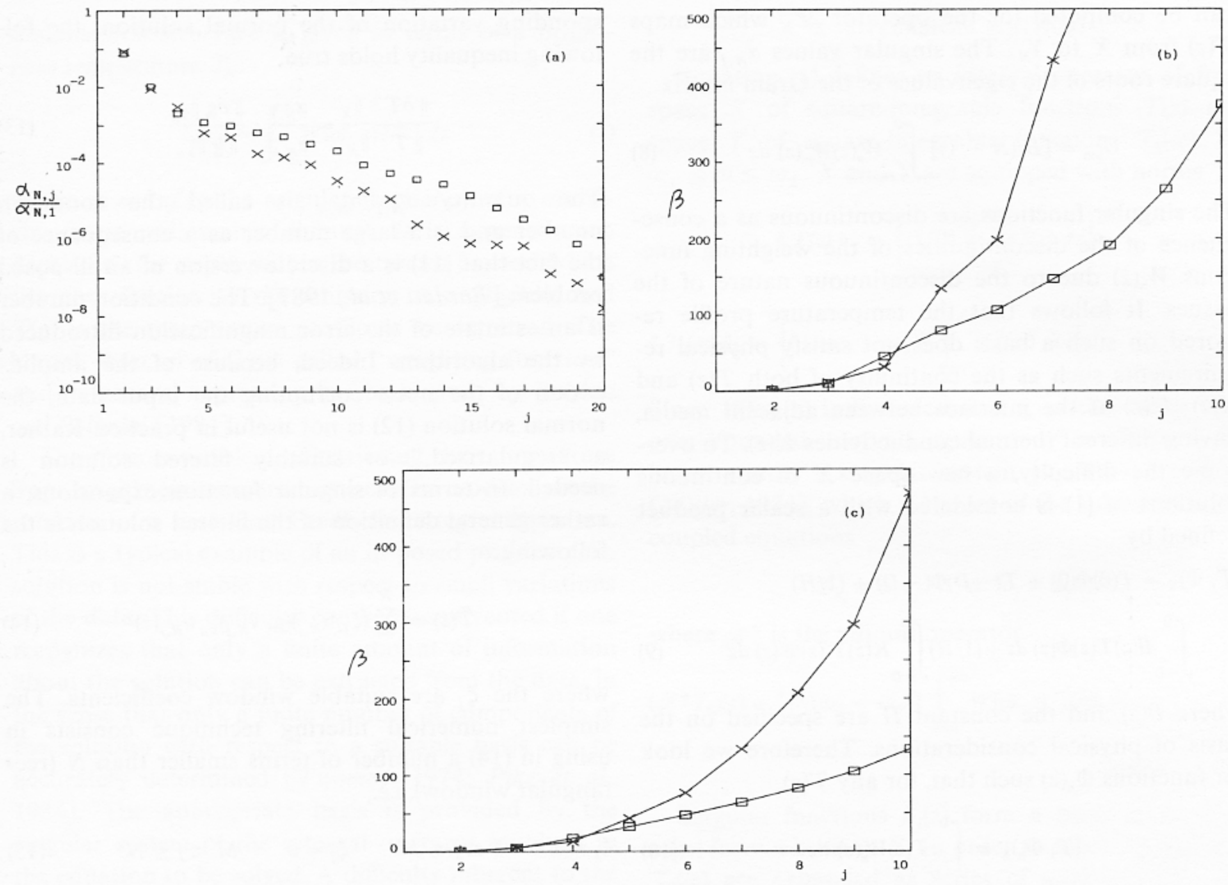

Fig. 1. (*a*) Ratio between *i*th and first singular value, (*b*) average error magnification for square, and (*c*) triangular filters. Squares refer to skin (2 mm), fat (4 mm), muscle structure; crosses refer to muscle (40 mm), bone (10 mm), muscle structure.

alternative expression

$$\beta = \{(1/N)\ \mathrm{Trace}[G]\}^{1/2} \cdot \left\{ \sum_{j=1}^{N} (\tilde{\xi}_j^2/\alpha_{N,j}^2) \right\}^{1/2} \tag{17}$$

where

$$\tilde{\xi}_j = \xi_j \bigg/ \left\{ \sum_{m=1}^{N} \xi_m^2 \right\}^{1/2} \tag{18}$$

can be used in place of the condition number to obtain a generally more realistic estimate of the noise magnification.

3. NUMERICAL SIMULATION AND DISCUSSION

A numerical simulation has been conducted for two arrangements of tissues, skin-fat-muscle (SFM) and muscle-bone-muscle (MBM). The number of degrees of freedom (NDF) of both structures has been evaluated. This number has practical relevance, since it represents the minimum number of measurements that must be performed without loosing information about the radiometric integral equation for a given signal-to-noise ratio. A convenient way to evaluate the NDF consists in examining the ratio between the *i*th and the first singular value. According to (13), this parameter gives an estimate of the NDF, once the noise-to-signal ratio of the measurements is known. $N = 20$ monochromatic measurements in the frequency range 1.5 to 6.5 GHz have been assumed to approximate the continuous case. Figure 1*a*, which reports the condition numbers for the SFM and the MBM structures, indicates that an MBM arrangement possesses a higher NDF, and hence more accurate temperature reconstructions can be achieved in a muscle layer backed by a bone. The effect of filtering on the average error magnification can be appreciated from Figures 1*b* and 1*c*, which refer to square and triangular filters, respectively. The tri-

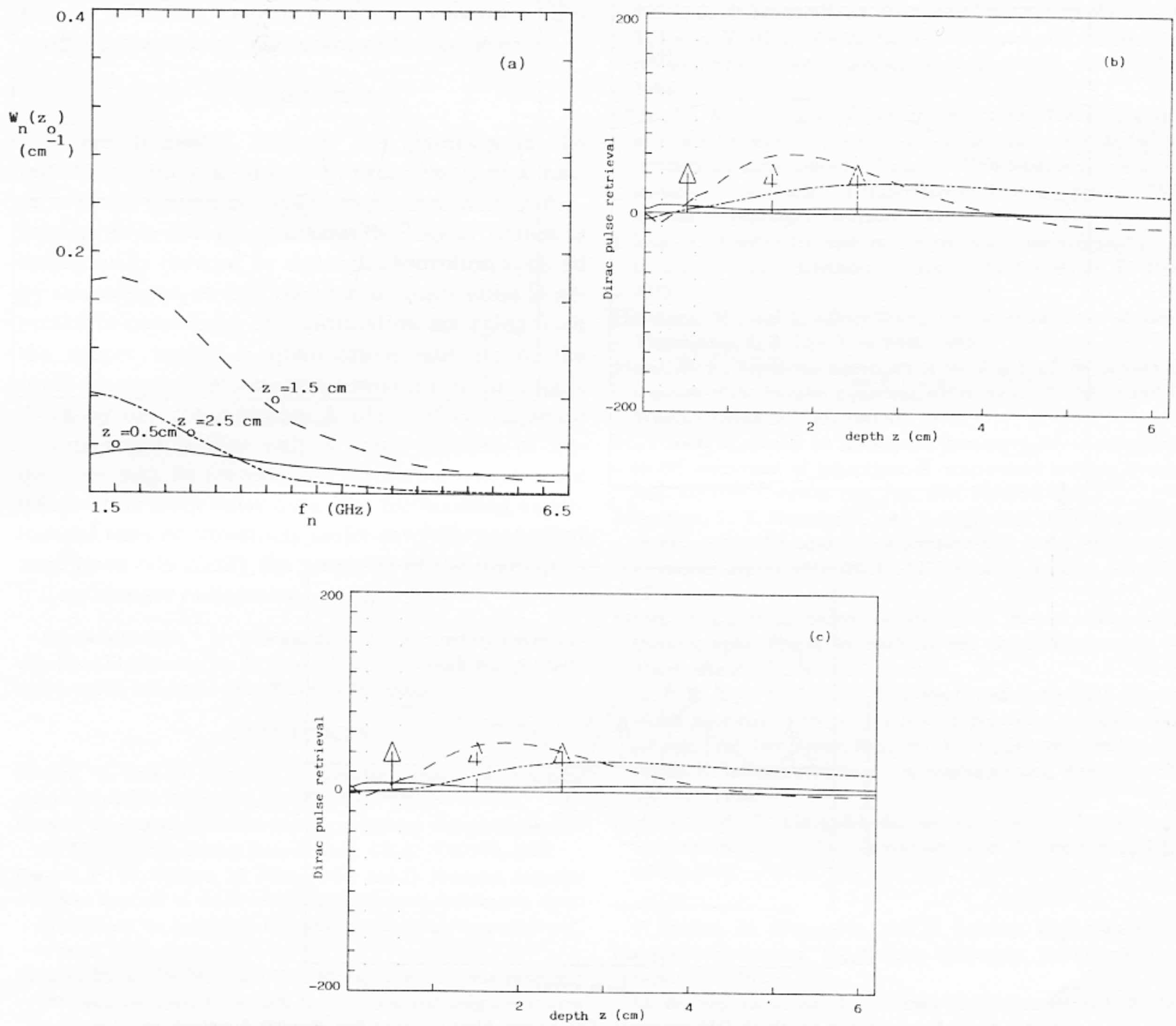

Fig. 2. Brightness measured by a radiometric channel at frequency f when Dirac pulses of temperature are placed at depths 5 mm (solid line), 15 mm (dashed line), and 25 mm (dotted-dashed line) (*a*) in an SFM structure, and (*b*) corresponding retrievals by use of square filter employing four singular functions, and (*c*) triangular filter employing six singular functions.

angular filter appears to yield retrievals more robust with respect to noise in the input data, that is, allows the use of a larger number of terms in the solution (14).

The performance of the reconstruction algorithm has been tested by assuming the temperature distribution to be Dirac pulses placed at different depths inside the tissues. The radiometric data as a function of frequency, i.e., the brightness temperature that would be measured by a radiometric channel at frequency $f = \omega/2\pi$ is reported in Figure 2*a* for an SFM structure and in Figure 3*a* for an MBM structure. From (1) we note that, if the Dirac pulse is at z_0, then the data are simply the values of $W_n(z_0)$. These radiometric data have been used as an input to the reconstruction algorithm. The restored temperature distribution has the meaning of a global pulse response function of the system consisting of the radiometer and of the retrieval algorithm. In practice such a function provides a measure of the accuracy and the resolution in the localization of sharp "thermal anomalies." Figures 2*b* and 2*c* show the re-

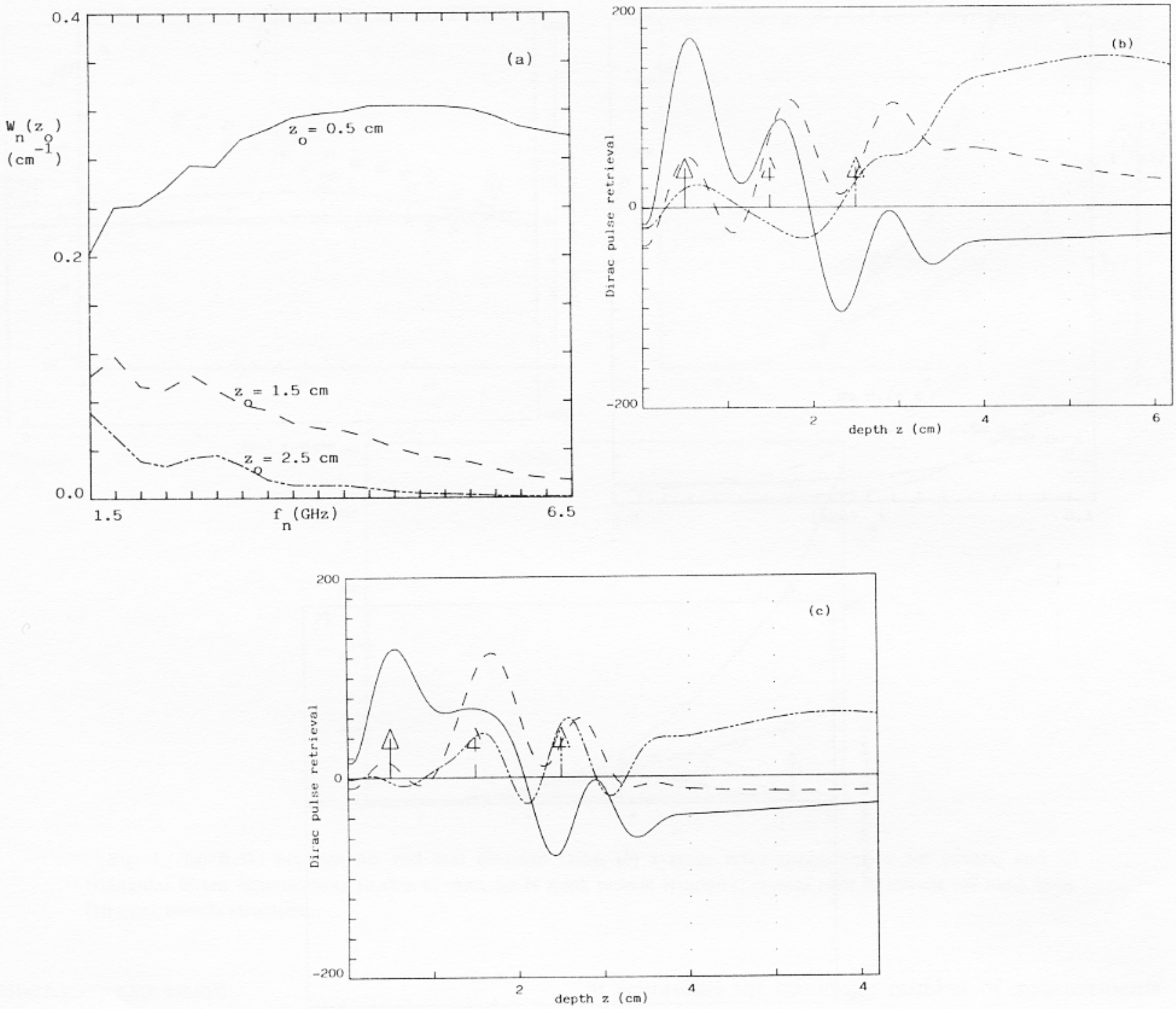

Fig. 3. As in Figure 2, but for the MBM structure. The square filter employs five singular functions, and the triangular filter employs nine singular functions.

trievals obtained in the SFM structure by using square and triangular windowing, respectively. Figures 3*b* and 3*c* refer to the corresponding retrievals for the MBM case. The number M of terms to use in (14) has been determined from Figure 1, assuming a 1% noise-to-signal ratio. For a given depth of the pulse, the reconstructions are more effective in this latter case of muscle backed by bone, as could be expected from the higher number of degrees of freedom which this structure possesses. The quality of the reconstructions obtained by the triangular filter appears to be superior to that of the square filter. This is a consequence of the higher number of terms that, for a given noise-to-signal ratio, can be used. According to Figure 1, the number of terms to use in the reconstruction decreases with increasing noise. This, again, degrades the retrieval accuracy, specially when the thermal impulse is deep in the tissues. The general trend of the result degradation with noise corruption is essentially similar to that appearing from Figures 3*b* and 3*c*. In the above reconstruction procedure the weighting functions are assumed known. In practice this may not be the case, since the assumed geometric and/or electric parameters of the tissues may differ from the true ones. This additional source of error, together with the modeling difficulties, in particular cases is expected to increase substantially the overall inaccuracy of the retrievals.

4. CONCLUSIONS

Severe difficulties arise in the inversion of the radiometric integral equation to retrieve thermal patterns in the human body. The thermal radiation originating from the subcutaneous biological tissues is dramatically reduced by the high absorption suffered by microwaves, so that the measurement noise is expected to overwhelm the information emerging from the deeper layers. A quantitative estimate of the small amount of information contained in the data is given by our computation. A practical consequence of this result is that only a small number of frequencies may be required for extracting temperature information from noisy data. The forthcoming experimental tests on phantoms under carefully controlled conditions will clarify the potential of the multispectral microwave radiometry.

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