Earth Observation Laboratory
PhD Program in GeoInformation
DISP - Tor Vergata University

Velocity vector estimation of moving targets from SAR images

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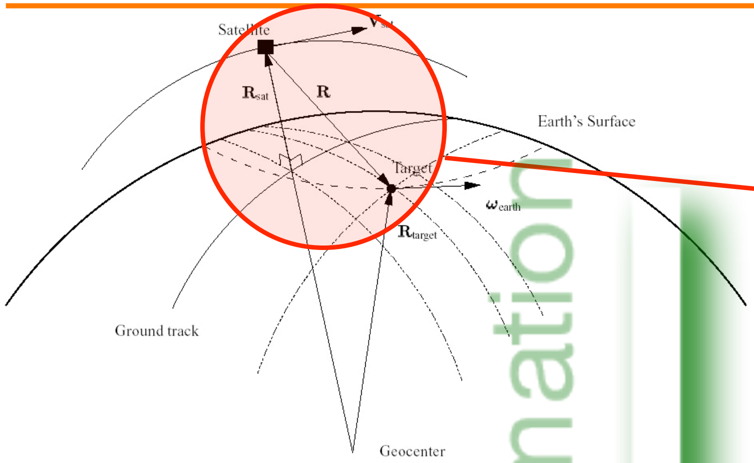
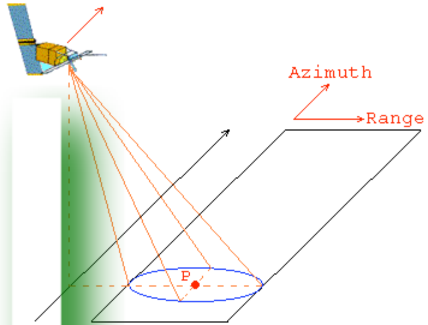
THESIS ORGANIZATION

1) Analysis of SAR theory and effects of the motion in SAR images
2) Analysis of the developed raw data simulator for moving target
3) Velocity estimation algorithm to derive the motion parameters from two-channel raw data
4) Analysis on the possibility to apply the velocity estimation algorithm to single channel SAR with sub-aperture processing
5) Velocity estimation algorithm from amplitude data

MAIN CONTRIBUTIONS

1) Development of a SAR raw data simulator for MTI applications
2) Development of a velocity estimation algorithm which derives the motion parameters from two-channel raw data without a priori information
3) Theoretical demonstration of the information type which can be obtained from a splitted single aperture
4) Development of a velocity estimation algorithm working on the amplitude images without a priori information





The SAR signal is scattered on the bi-dimensional range-azimuth plane

$$\left| t - 2\frac{R(s)}{c} \right| \leq \frac{\tau}{2}$$

IRF of the SAR: $$h(s,t \mid s_c, R_c) = \cos\left\{ 2\pi \left[f_c \left(t - 2\frac{R(s)}{c} \right) + \frac{K}{2} \left(t - 2\frac{R(s)}{c} \right)^2 \right] \right\}$$

Fast time (range time)

Slow time (azimuth time)

$$\left| s - s_c \right| \leq \frac{S}{2}$$

SAR processing: compression that focuses the IRF around a point

$$\zeta(R_0) = \int_{-\infty}^{+\infty} h^{-1}(R_o \mid R) v_r(R) dR$$

We consider an algorithm of the first class that works in the time domain, the so-called Time Domain Correlation (TDC)

The algorithm is divided into 3 major steps:

1) Range compression;
2) Range migration compensation;
3) Azimuth compression for every pixel in the time domain.

SAR PROCESSING THEORY

Transmitted chirp signal

$$s(t) = \cos\left[2\pi\left(f_c t + K\frac{t^2}{2}\right)\right]$$

Base-band signal of the received data:

$$\hat{v}_r(s,t) = 0{,}5\exp\left[-j4\pi\frac{R(s)}{\lambda}\right]\cdot\exp\left\{j\pi K\left[t - 2\frac{R(s)}{c}\right]^2\right\}$$

Range compression:

$$g(s,t) = \int_{t-\frac{\tau}{2}}^{t+\frac{\tau}{2}} \hat{v}_r(s,t')s^*(t'-t)dt' = B\exp\left(-j4\pi\frac{R(s)}{\lambda}\right)\mathrm{sinc}\left\{\pi B\left[t - 2\frac{R(s)}{c}\right]\right\}$$

$$s(t) = 0.5\exp(j\pi K t^2)$$

Range migration compensation:

$$\hat{g}(s_k, R(s_k)) = \sum_{i=-a}^{+b}\hat{g}(s_k, R_i)\mathrm{sinc}\left[\pi f_s\left(\frac{2\cdot(R(s_k)-R_c)}{c} - \frac{i}{f_s}\right)\right]$$

Azimuth compression:

$$\zeta(s_c', s_c, R_c) = \int_{s_c-S/2}^{s_c+S/2}\hat{g}(s|s_c,R_c)h_{az}^{-1}(s-s_c'|s_c,R_c)ds = S\cdot\mathrm{sinc}\left[\pi\frac{2V_{st}^2}{\lambda R_c}S(s_c'-s_c)\right]$$

$$h_{az}^{-1}(s-s_c|s_c,R_c) = \exp\left\{j\frac{4\pi}{\lambda}[R(s)-R_c]\right\}$$

Range equation:

$$R^2(s) = R_c^2 + [V_{st}(s-s_c)]^2$$

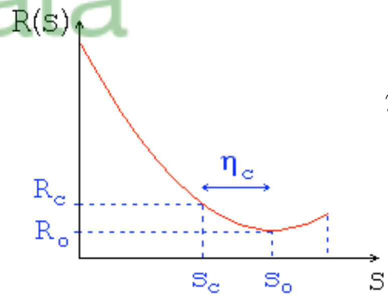
Parabolic approximation:

$$R(s) = R_c - \lambda\frac{f_{Dc}}{2}(s-s_c) - \lambda\frac{f_R}{2}\frac{(s-s_c)^2}{2}$$

Notation:

Doppler Centroid processing: Azimuth time $s - s_c$

Zero Doppler processing: Azimuth time $\eta = s - s_0$

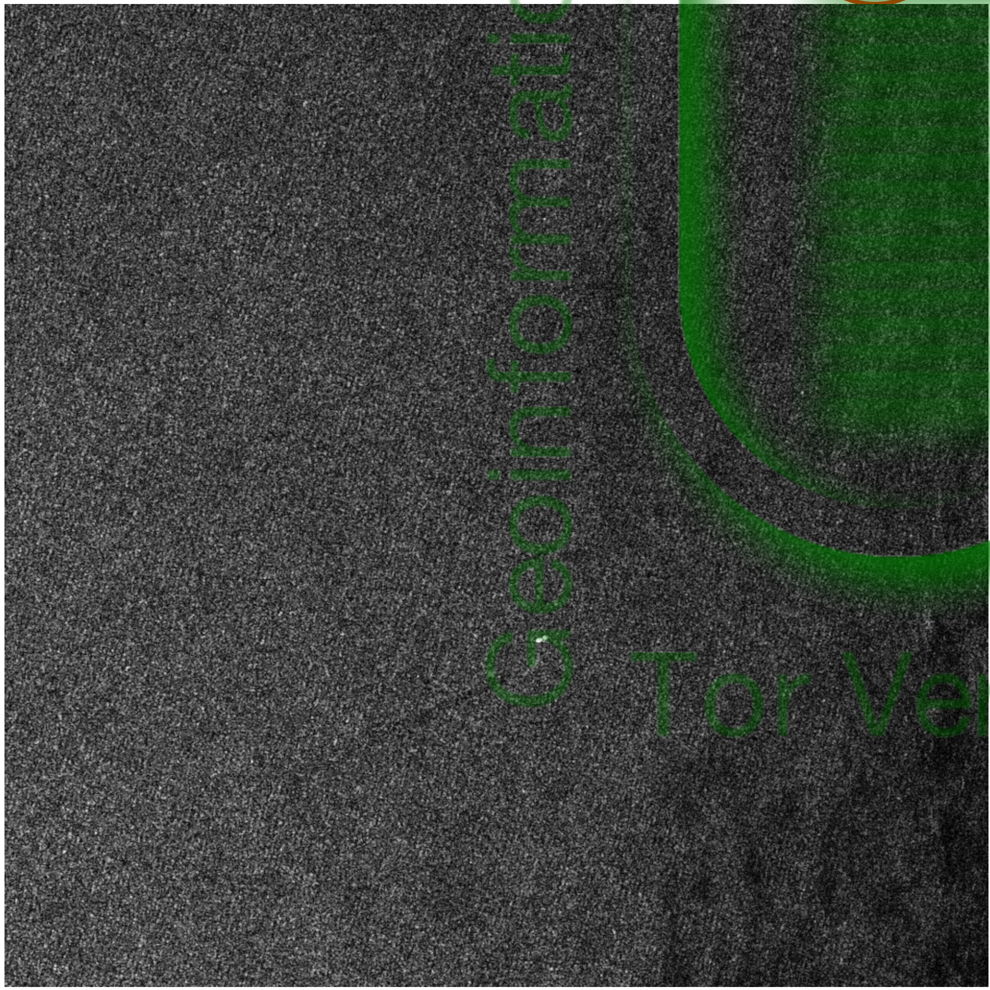


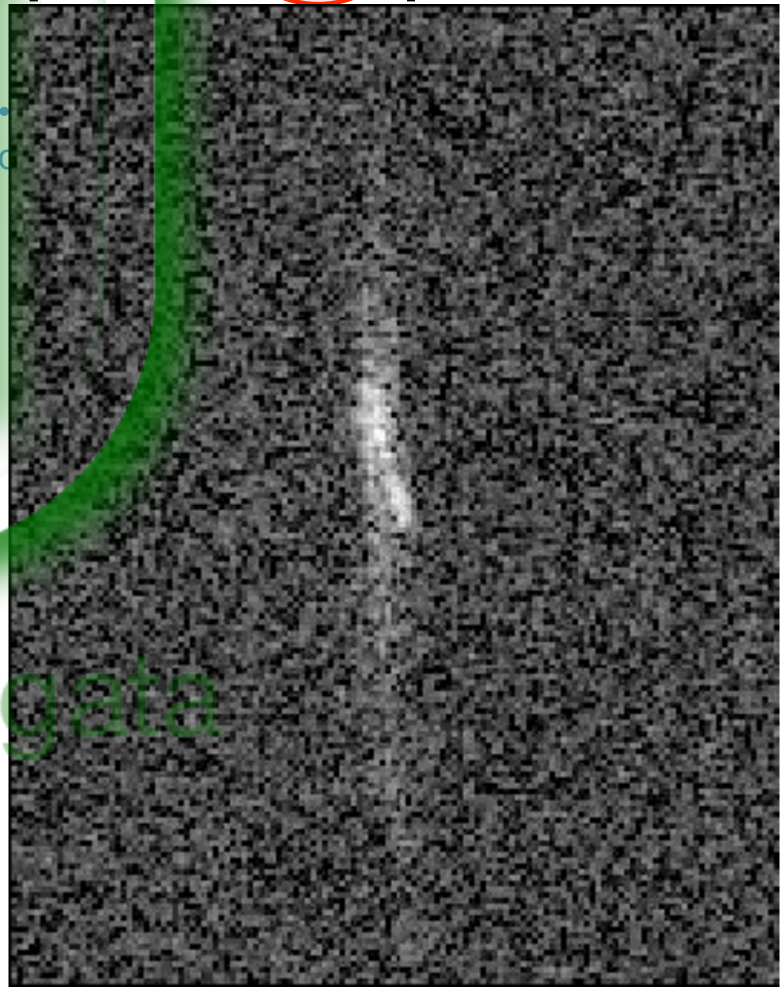
$$\eta_c = s_c - s_0 = \frac{f_{Dc}}{f_R}$$

MOTION EFFECTS IN SAR IMAGES

Slant range for stationary target: $R(\eta) \approx R_0 + \frac{V_{sat}^2}{2R_0}\eta^2 = R_0 + -\frac{\lambda f_R}{4}\eta^2$

Slant range for moving target: $R(\eta) \approx R_0 + \frac{y_0 v_{rg}}{R_0}\eta + \frac{1}{2R_0}\left[\left(v_{az} - V_{sat}\right)^2 + v_{rg}^2\left(1 - \frac{y_0^2}{R_0^2}\right)\right]\eta^2 \quad y_0 = R_0 \cdot \sin\vartheta$





RAW DATA SIMULATOR

The (IRF) is characterized by the received voltage and the antenna directivity

- Received voltage in baseband

$$\hat{v}_r(\eta, t \mid \eta_c, t_0) = 0,5\exp\left[-j\Phi(\eta)\right]\cdot\exp\left\{j\pi K\left[t - 2\frac{R(\eta)}{c}\right]^2\right\} \qquad \left|t - 2\frac{R(\eta)}{c}\right| \le \frac{\tau}{2} \qquad \left|\eta - \eta_c\right| \le \frac{S}{2}$$

$$\Phi(\eta) = \frac{4\pi}{\lambda}R(\eta)$$

Key of the simulator and of the SAR processor

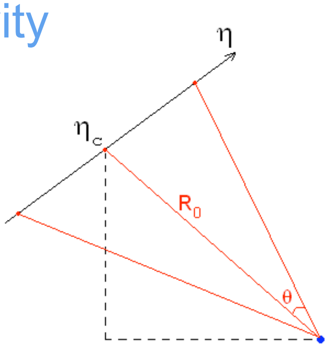
- Antenna directivity

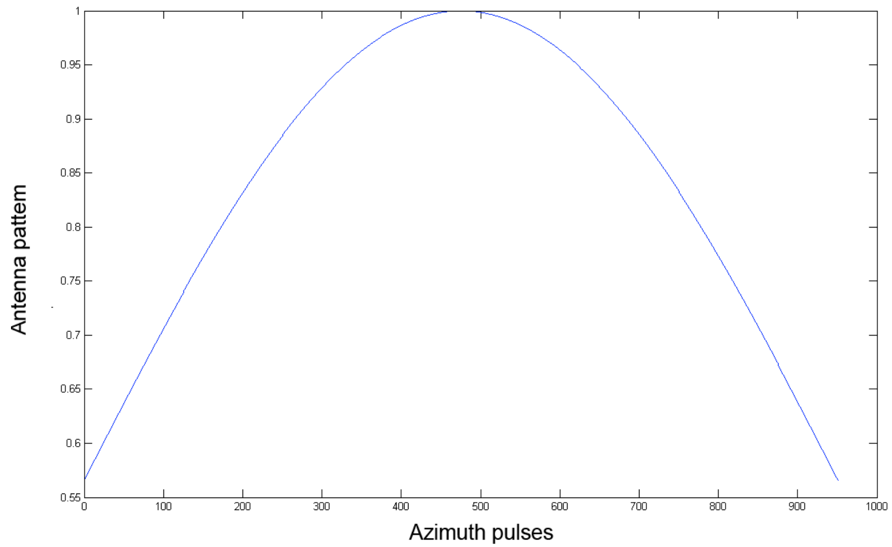
$$D(\theta) = \mathrm{sinc}^2\left[\frac{\mathrm{L_a}}{\lambda}\cdot\sin\theta\right]$$

θ very small → $\sin\theta \approx \theta$

$$\theta = tg^{-1}\left(\frac{\eta - \eta_c}{R_0}\right)$$

→ $$D(\eta \mid \eta_c) = \mathrm{sinc}^2\left[\frac{\mathrm{L_a}}{\lambda}\cdot tg^{-1}\left(\frac{\eta - \eta_c}{R_0}\right)\right]$$





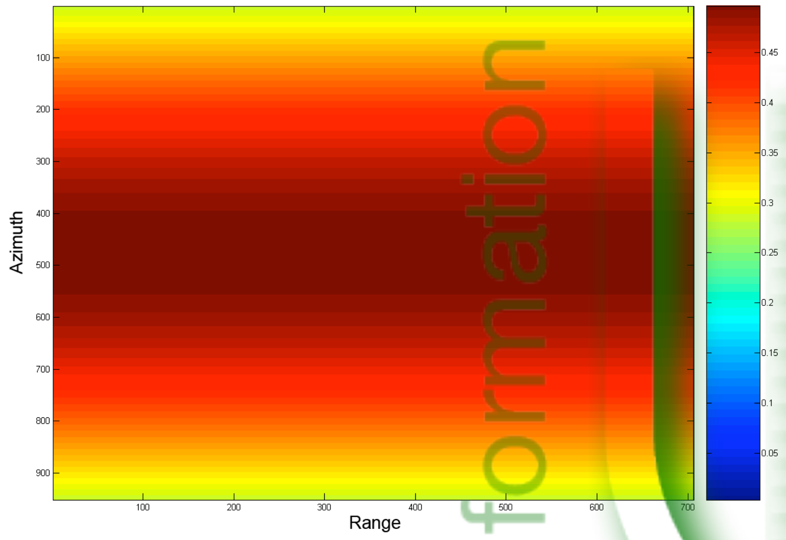
Impulse function in each pixel $$h_{SAR}(\eta, t \mid \eta_c, t_0) = D(\eta \mid \eta_c)\cdot\hat{v}_r(\eta, t \mid \eta_c, t_0)$$

Sum of all the voltage contributions coming from the pixel provides raw data

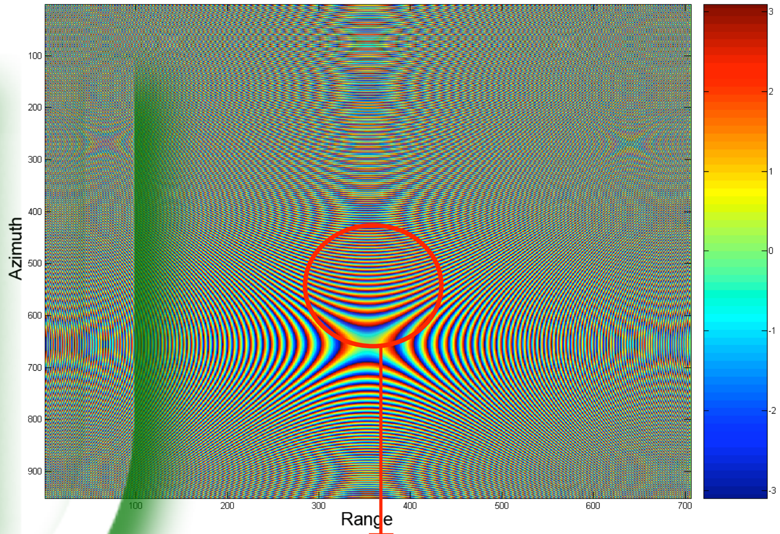
$$V = \sum_{\eta_c}\sum_{t_0} h_{SAR}(\eta, t \mid \eta_c, t_0)$$

RAW DATA SIMULATOR

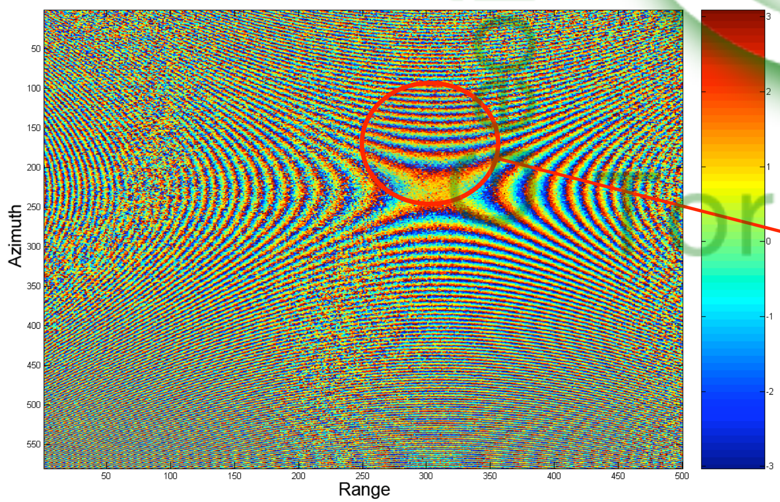
Amplitude raw data



Phase raw data



Hyperbolic form of the phase contour caused from the range equation



Degradation of the Hyperbolic iso-phase for the noise

VELOCITY ESTIMATION ALGORITHM

1. The algorithm retrieves the two velocity components of moving targets from two-channel raw data;
2. The coupling of range and azimuth velocity is taken in account

$$R_{mov}(\eta) \approx a + b\left(v_{rg}\right)\eta + c\left(v_{az}\right)\eta^2$$ Classical mode: both components are decoupled

$$R_{mov}(\eta) \approx a + b\left(v_{rg}\right)\eta + c\left(v_{rg}, v_{az}\right)\eta^2$$ Algorithm: considers the coupling in the "c" term

The estimation of the linear term *a is not possible if the quadratic term is not compensated.*

But the estimation of the quadratic term needs the compensation of the linear term!

USED VELOCITY ESTIMATION TECHNIQUES

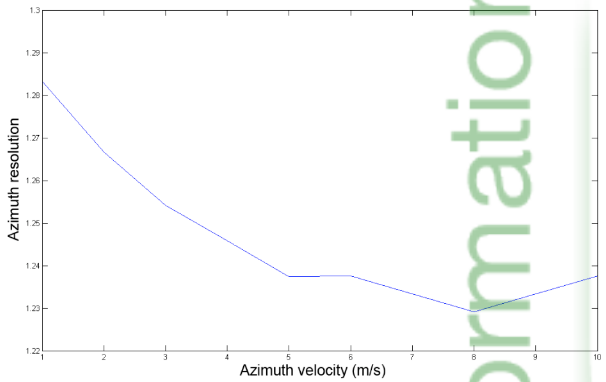
To derive the full velocity vector is necessary to use amplitude and phase information

1. Azimuth velocity: use of an azimuth filters bank (amplitude information)
2. Range velocity: use of the Along Track Interferometry (ATI) with two channels (phase information)

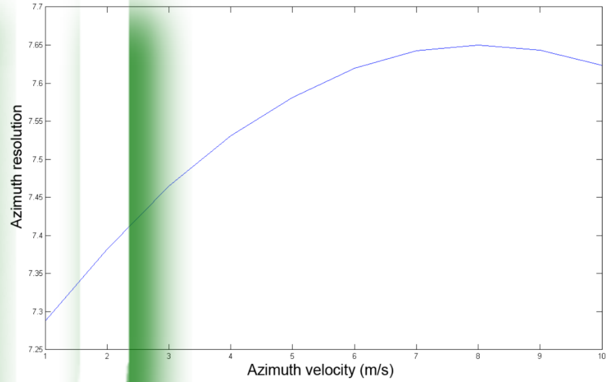
AZIMUTH VELOCITY FILTER BANK

The raw data are focused using a bank of azimuth filters; the analysis of the IRF allows to estimate the azimuth velocity, matching the right filter

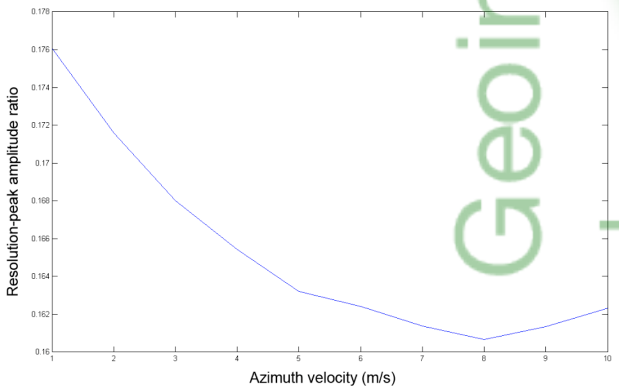
Maximization of the mainlobe amplitude



Maximization of the azimuth resolution

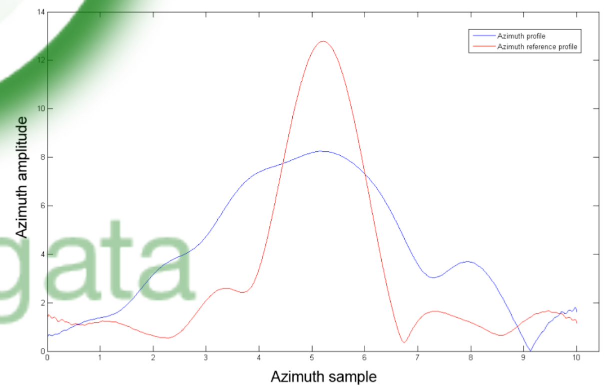


Maximization of the azimuth resolution-amplitude ratio



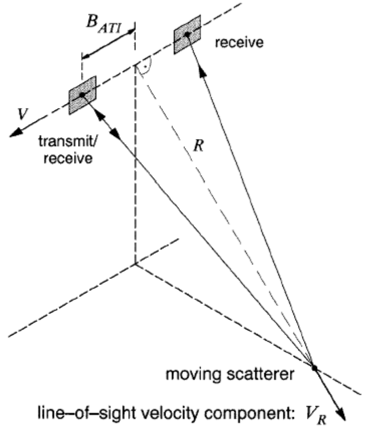
Maximization of the correlation coefficient between moving target IRF and reference stationary IRF

$\rho = 0,9998$



To recontruct the IRF in strong noise, the profile is approximated to the Gaussian function that is better correlated with the reference signal reconstruction

In two-channel SAR system the antennas along the flight-line are separated by a spatial baseline



ATI signal:

$$ATI = s_1(\eta,t)s_2^*(\eta,t) = |s_1(\eta,t)||s_2(\eta,t)| \cdot \exp\left(j\left[\varphi_1(\eta,t) - \varphi_2(\eta,t)\right]\right)$$

ATI phase:

$$\Phi(\eta,t) = \frac{4\pi}{\lambda}\frac{B_{ATI}v_{sr}}{V_{sat}} + \frac{2\pi}{\lambda}B_{ATI}\left(\frac{1}{V_{sat}} + \frac{v_{az} - V_{sat}}{V_{st}^2}\right)\Delta v_{sr} - \frac{2\pi}{\lambda}\frac{v_{az}B_{ATI}}{R_0}\eta_c$$

Useful informative content to derive the radial velocity

Term negligible, because

$$\frac{v_{az} - V_{sat}}{V_{st}^2} \approx -\frac{1}{V_{sat}}$$

Error term caused by an un-symmetrical target trajectory around the broadside time, by non-symmetries caused by the RCS variation during the integration time (i.e.: for the changing of aspect angle) and by a not compensation of the azimuth velocity

METHODOLOGY

1° STEP

$\hat{g}(\eta,t)$ → Azimuth compression →

$$h_{az}^{-1}(\eta,\Delta v_{az}) = \exp\left\{ j\frac{4\pi}{\lambda}\cdot\frac{(\Delta v_{az}-V_{sat})^2}{2R_0}\eta^2 \right\}$$

The right velocity is selected from the filter that maximizes the ratio between azimuth resolution and mainlobe amplitude of the IRF

$$\Delta v_{az} \in \left[-v_{az}^{\max}, +v_{az}^{\max} \right]$$

Filter separation:

2° STEP

$\hat{g}(\eta,t\,|\,v_{az}^{'})$ → Azimuth compression → 1° antenna / 2° antenna → ATI →

$$h_{az}^{-1}(\eta,v_{az}^{'}) = \exp\left\{ j\frac{4\pi}{\lambda}\cdot\frac{(v_{az}^{'}-V_{sat})^2}{2R_0}\eta^2 \right\}$$

A range velocity filter bank is used to refine the estimation

$$\Delta v_{az} \in \left[-0,24\cdot v_{rg}^{'}, +0,24\cdot v_{rg}^{'} \right]$$

Filter separation:

3° STEP

$\hat{g}(\eta,t)$ → Azimuth compression →

We maximize the correlation coefficient between moving target IRF and reference IRF with the profiles rounded as gaussian

$$h_{az}^{-1}(\eta,v_{rg},v_{az}) = \exp\left\{ j\frac{4\pi}{\lambda}\left[\frac{y_0 v_{rg}}{R_0}\eta + \frac{1}{2R_0}\left(\left(\Delta v_{az}+v_{az}^{'}-V_{sat}\right)^2 + v_{rg}^2\left(1-\frac{y_0^2}{R_0^2}\right)\right)\eta^2 \right] \right\}$$

Selection criterion more sensitive to little velocity variation

$$\Delta v_{az} \in \left[-0,25\cdot v_{az}^{'}, +0,25\cdot v_{az}^{'} \right]$$

Filter separation:

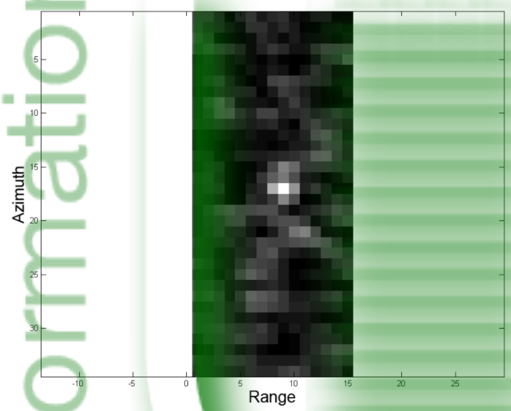
SEA BACKGROUND FOR SHIP MONITORING

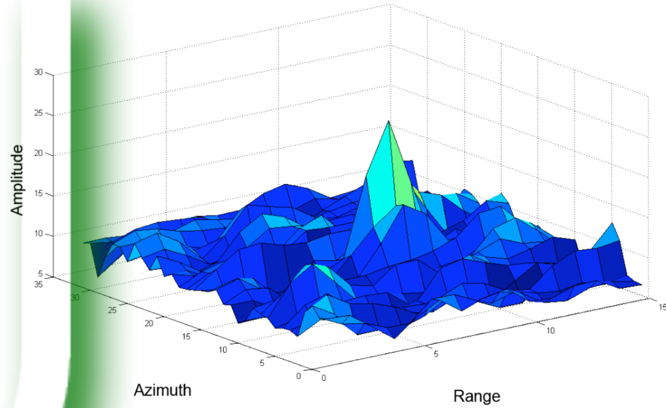
Gamma distribution

$$f(x|a,b)=\frac{1}{b^{a}\Gamma(a)}\cdot x^{a-1}\exp\left(-\frac{x}{b}\right)$$

$$\Gamma(a)=\int_0^{\infty}\exp(-t)\cdot t^{a-1}dt$$

SCR = 18 dB, variance = 2 dB



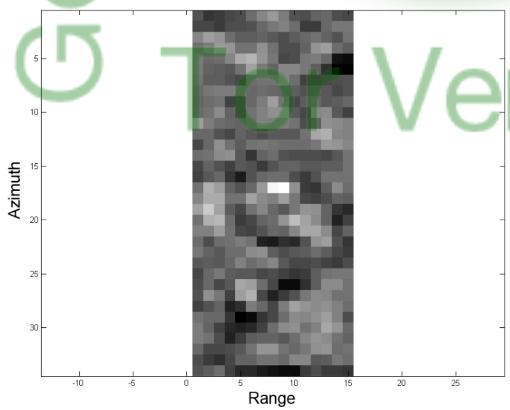


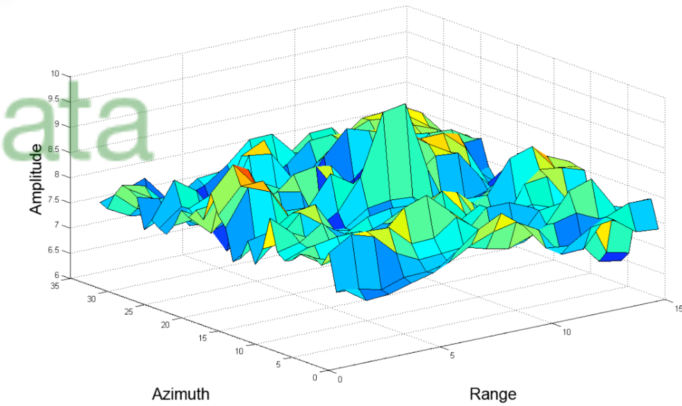
SHRUBS BACKGROUND FOR CAR TRAFFIC MONITORING

Rayleigh distribution

$$f(x|\sigma)=\frac{x}{\sigma^{2}}\cdot\exp\left(-\frac{x^{2}}{2\sigma^{2}}\right)$$

SCR = 9 dB, variance = 3,1 dB





SIMULATION: RESULTS

To obtain the statistical parameters of the estimations:

1. We choose a velocity vector
2. We varied the range velocity within a little interval
3. We varied the azimuth velocity within a little interval

Difficulty to estimate the low azimuth velocity

Background	Range error mean	Azimuth error mean	Range standard deviation	Azimuth standard deviation
Constant (20 dB)	6,5%	7,5%	0,1 m/s	2,2 m/s
Sea	21,9%	12,3%	0,3 m/s	3,3 m/s
Shrubs	35%	25%	3 m/s	7,1 m/s

The ATI can fail for low SCR (<15 dB)

For high range velocity the signal energy of the moving target might be shifted outside the azimuth processed bandwidth

Difficulty to reconstruct the azimuth profile

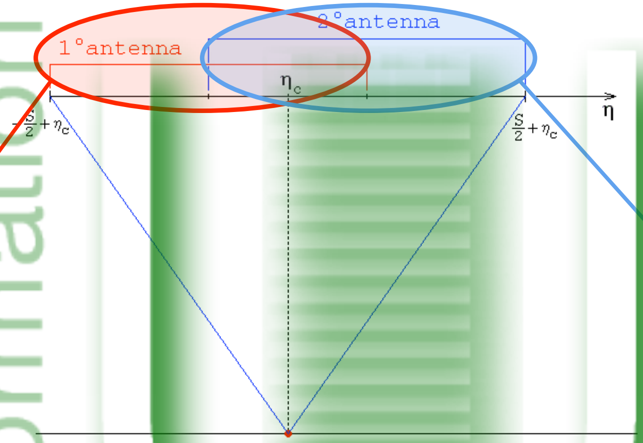
CONSIDERATIONS

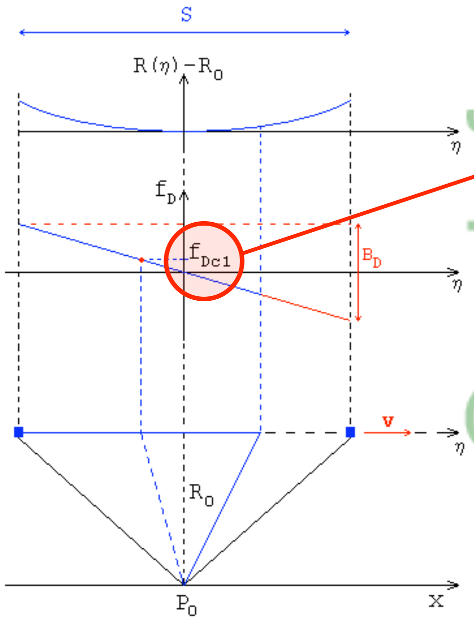
1. MTI applications need high resolution and high SCR
2. We simulated the parameters characteristics of ERS with the intent to apply the algorithm on single channel SAR → starting disadvantageous situation
3. We work without *a priori* information
4. The ATI suffers low SCR

For TerraSAR-X, with SCR=5dB the standard deviation of the derived range velocity is 30 km/h

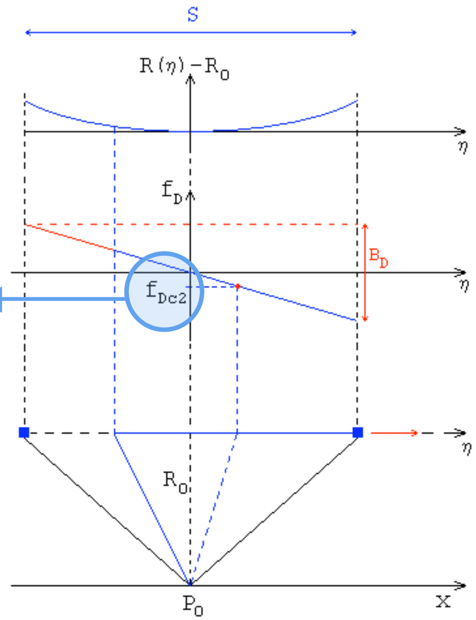
ADAPTATION FOR SINGLE CHANNEL SAR

Aim: simulate a two channel SAR by generating two sub-apertures of single channel





$$f_{Dc1} = f_R \cdot \left(\frac{\eta_{ap}}{2} \cdot S - \frac{S}{2} \right)$$



$$f_{Dc2} = -f_R \cdot \left(\frac{\eta_{ap}}{2} \cdot S - \frac{S}{2} \right) = -f_{Dc1}$$

Obtained from:

Time-frequency relation $\eta - \eta_c = \frac{1}{f_R} f_D$

$$f_D = f_R (\eta - \eta_c)$$

$$\pm \frac{S}{2} = \mp \frac{1}{f_R} \cdot \frac{B_D}{2} \quad \text{for} \quad \eta = \pm \frac{S}{2} + \eta_c$$

SUB-APERTURE PROCESSING

PRE - FILTERING

Band-pass FIR filter:

$$h_D(n) = \frac{B_{D\_look}}{PRF} \cdot \mathrm{sinc}\left(\frac{\pi n \cdot B_{D\_look}}{PRF}\right) \cdot \left[0,54 - 0,46 \cdot \cos\left(\frac{\pi n}{N_{centro}}\right)\right] \cdot \exp\left[j 2\pi f_{Dci}\left(\frac{n}{PRF}\right)\right]$$

- FIR filter in time domain
- Hamming window to reduce the Gibbs phenomenon
- The filter is not in base-band

$$N_{centro} = N - \tfrac{1}{2}$$

N = filter length

SUB - APERTURE AZIMUTH COMPRESSION

Filtered range compressed data → Azimuth compression → 1° look, 2° look

$$h^{-1}_{az_i}\left(\eta \middle| \eta_c, R_c\right) = \exp\left\{-j 2\pi \left[f_{Dci}\eta + \frac{f_R \eta^2}{2}\right]\right\}$$

i = 1,2

$$f_{Dci} = \pm f_R \cdot \left(\frac{\eta_{ap}}{2} \cdot S - \frac{S}{2}\right)$$

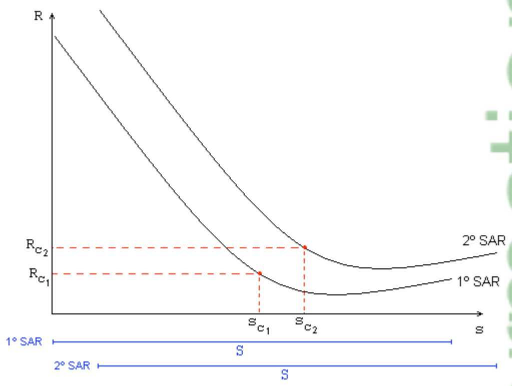
The looks are co-registrated

NOTE: the maximum non-ambiguous velocity depends on the global aperture fractional

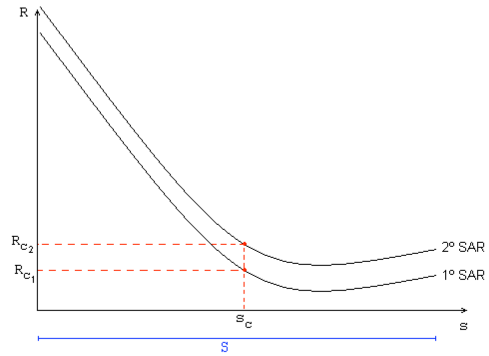
η_{ap} defines the baseline between the two antennas in ATI configuration

TWO CHANNEL

The antennas acquire two sets of data each with a different time centers R_{ci} .If the target moves in the range direction, the temporal difference corresponds to a shift of the slant range centers s_{ci}



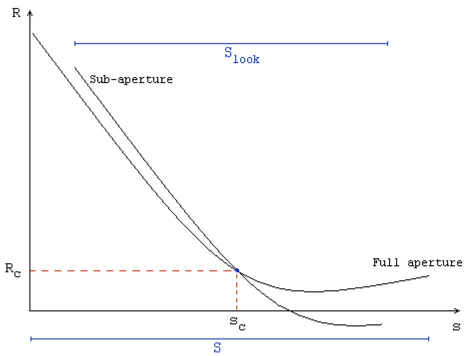
The co-registration operation of the channels superimposes the images, removing the time difference.

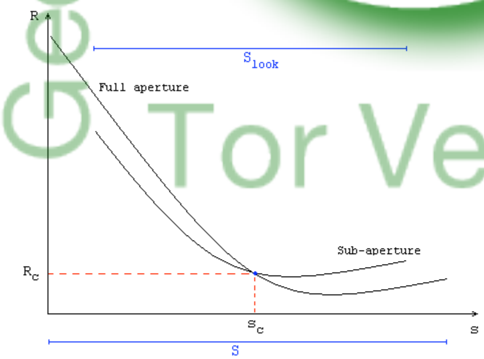


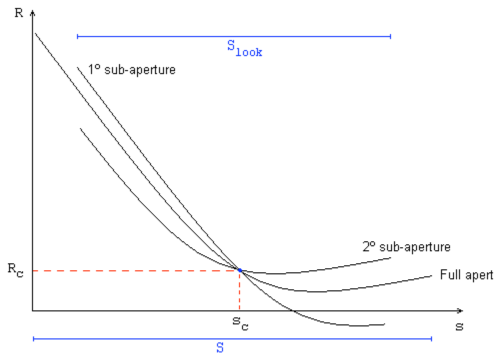
SINGLE CHANNEL

Splitting the antenna in two sub-apertures means to use each look to focus the pixel with coordinates (s_c, R_c)

This means to have a temporal baseline. Note: for each sub-aperture the Doppler history does not change







There is not a spatial diversity!

SAR PROCESSING BASIC THEORY

The target moves with radial velocity $v_{sr} = v_{rg} \cdot sen(\vartheta)$

Range migration: $$R_{mov}(s) = R_c - \lambda \frac{(f_{Dc} - v_{sr})}{2}(s - s_c) - \lambda \frac{f_R}{2} \frac{(s - s_c)^2}{2}$$

Range compressed data: $$\hat{g}(s,t) = B \exp\left(-j4\pi \frac{R_{mov}(s)}{\lambda}\right) \cdot \text{sinc}\left\{\pi B \left[t\text{-}2 \frac{R_{mov}(s)}{\text{c}}\right]\right\}$$

Azimuth compression: $$\zeta(s_c' | s_{ci}, R_c) = \int_{s_{ci} - S_i/2}^{s_{ci} + S_i/2} \hat{g}(s | s_{ci}, R_c) \cdot h_{az}^{-1}(s - s_{ci}' | s_{ci}, R_c) ds$$

Two errors are made, choosing a stationary matched filter and compensating the range migration as for stationary target

SUB-APERTURE PROCESSING

$$\zeta(s_c' | s_{c1}, R_c) = \int_{s_{c1} - S_1/2}^{s_{c1} + S_1/2} \hat{g}(s | s_{c1}, R_c) \cdot h_{az}^{-1}(s - s_{c1}' | s_{c1}, R_c) ds$$

$$\zeta(s_c' | s_{c2}, R_c) = \int_{s_{c2} - S_2/2}^{s_{c2} + S_2/2} \hat{g}(s | s_{c2}, R_c) \cdot h_{az}^{-1}(s - s_{c2}' | s_{c2}, R_c) ds$$

Compressed signal amplitude:

Compressed signal phase:

CONSIDERATION AND RESULTS VALIDATION

Compressed signal amplitude:

Compressed signal phase:

The phase is independent on the range velocity, only the amplitude peak shifts to the position $s_c' = s_{ci} - \frac{v_{sr}}{f_R}$

The ATI is not applicable on sub-aperture images to estimate the radial velocity!

To validate the theory we used a simulator, which works in steps:

1. Generate an azimuth chirp of a moving point-like target
2. Compensate the range migration for a stationary target
3. The range compressed data is filtered and compressed to obtain two symmetrical sub-apertures
4. The range velocity and the baseline are varied, to demonstrate that the phase and the amplitude level are independent of the velocity

We simulated the parameters characteristics of ERS

Percentage differential amplitude between the sub-apertures

Amplitude	Aperture look (%)	60	70	80	90
Velocity (m/s)					
0		0	0	0	0
5		1,3	0,36	0,03	0
10		2,6	0,69	0	0
15		3,8	0,86	0,03	0
20		5,1	0,79	0	0

Differential phase between the sub-apertures

Phase	Aperture look (%)	60	70	80	90
Velocity (m/s)					
0		0	0	0	0
5		π/113	< π/1000	< π/1000	< π/1000
10		π/58	< π/1000	< π/1000	< π/1000
15		π/39	< π/1000	< π/1000	< π/1000
20		π/30	< π/1000	< π/1000	< π/1000

The amplitude and the phase differential between the two channels shows a negligible increase with the velocity

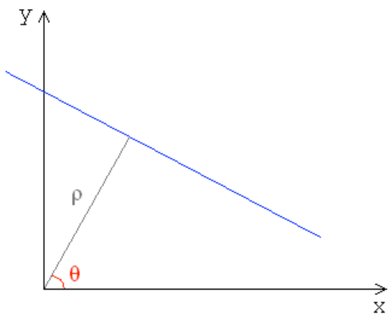
Note: errors for numerical approximations and for the use of a finite-length FIR filter

We propose an algorithm which estimates the full velocity vector of the ships from amplitude images, more easily available, without *a priori* information, using the Radon Transform (RT). It is very light from the computational point of view

RADON TRANSFORM

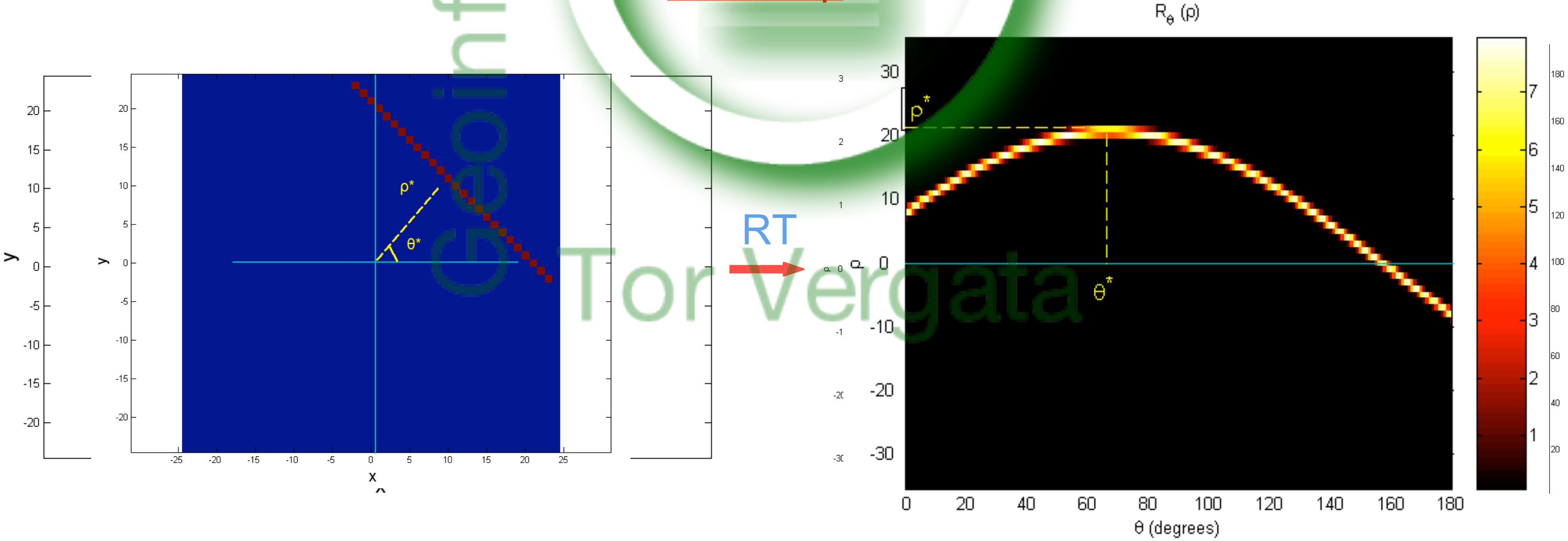
Given an image g in the coordinate system (x,y), the Radon transform is defined as

$$\breve{g}(\rho,\theta)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)\,\delta(\rho - x\cos\theta - y\sin\theta)\,dxdy \qquad \begin{cases} 0 \le \theta < \pi \\ -\rho_{\max} \le \rho < \rho_{\max} \end{cases}$$



The RT computes the projection of an image along the direction given by (ρ,θ), calculating the line integrals of the image along this direction.

RT of a line
RT of a point source



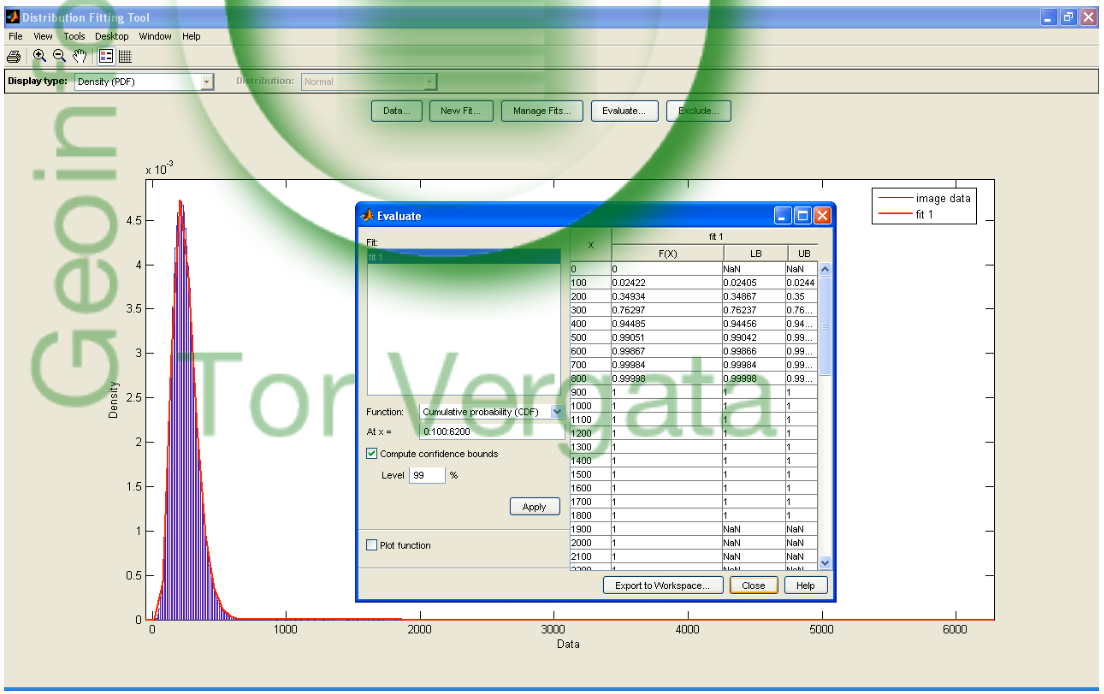
METHODOLOGY SCHEME

Image analysis → Ship detection → Ship focusing → Velocity estimation

ANALYSIS

Land masking → Division in sub-images → Histogram generation → Histogram fitting → Threshold selection

The gamma function fits very well the sea histogram



METHODOLOGY SCHEME

SHIP DETECTION

Threshold → CFAR

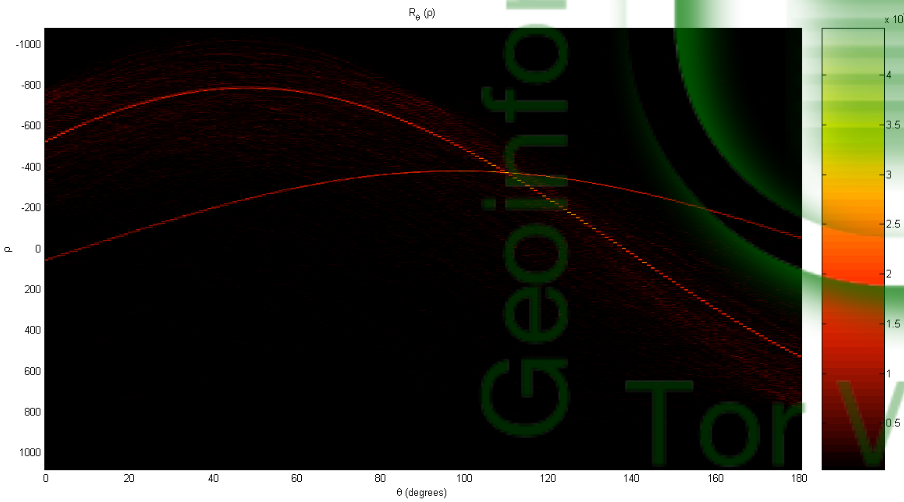
Image → CFAR → Ship map → Ship number / Coordinates / Length

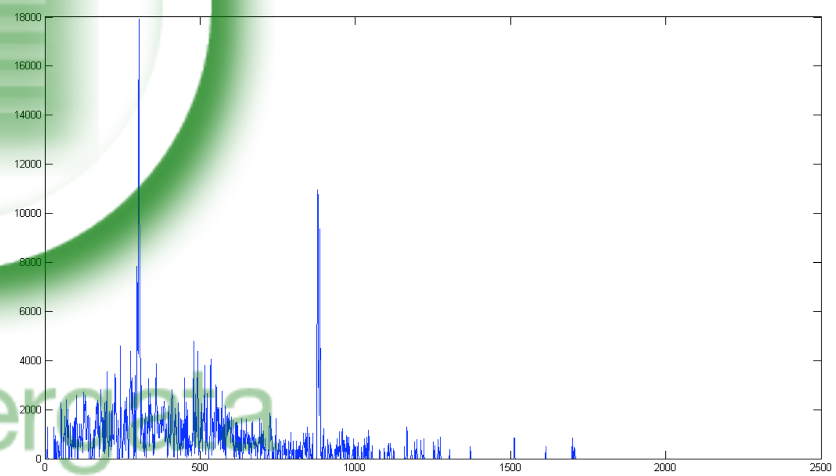
Ship map: composed by pixels with a binary value 1 if the pixel is classified as ship, 0 otherwise

The ship is a complex object, and can appear like different targets

To avoid this phenomenon, for every pixel classified like ship the algorithm sets to 1 the adjacent pixels

The estimated number of ship is verified using the RT, analyzing a profile of the RT for a constant angle





SHIP FOCUSING

Every ship is focused at the center of a sub-image

METHODOLOGY SCHEME

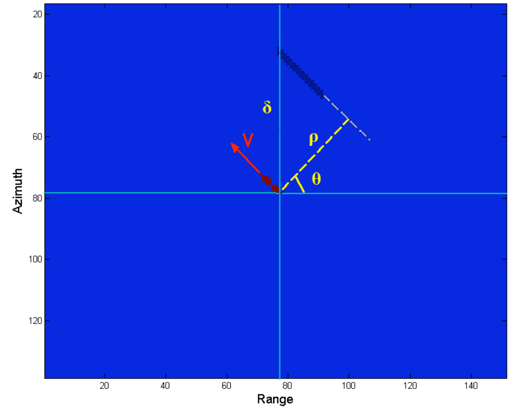
VELOCITY ESTIMATION

WOVE algorithm (from Wake Orientation to Velocity Estimation)

Pre-processing: Ship masking → High pass filter → Image logarithm → Image negative

Because the ship is on the center, RT estimates at the same time the angle of the wake orientation and the azimuth shift between ship and wake

RT determines the couple (ρ,θ) related to the velocity components



$$\rho = \delta \sin\theta \Rightarrow \delta = \frac{\rho}{\sin\theta}$$

Temporal shift $\eta_{shift} = \frac{2v_{sr}}{\lambda \cdot f_R}$

Spatial shift $\delta = v_B \cdot \eta_{shift}$

$$v_{gr} = \frac{\lambda f_R \delta}{2v_B} \cdot \frac{1}{\sin\phi}$$

[illegible]

SOVE algorithm (from Ship Orientation to Velocity Estimation)

Because the ship has an extended form, it can be considered like a line

1. The ship is focused at a known azimuth distance from the center.
2. With RT we estimate the orientation θ, taking the maximum of the image in the Radon domain
3. Finally the ship is focused at the center of the scene and the RT is applied to the image for the angle θ, to scan the space in the right direction
4. The velocity components are derived as in the WOVE algorithm

To resolve the ambiguity of the orientation estimate we scan all the space separately in the interval [0,π] and [π,2π].

RESULTS

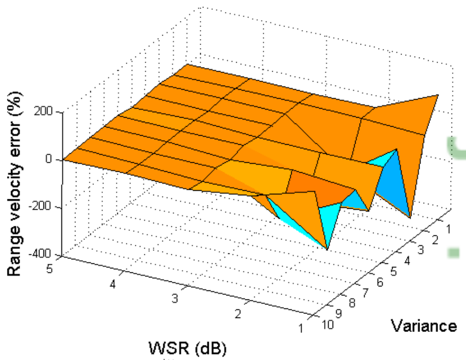
SIMULATION RESULTS

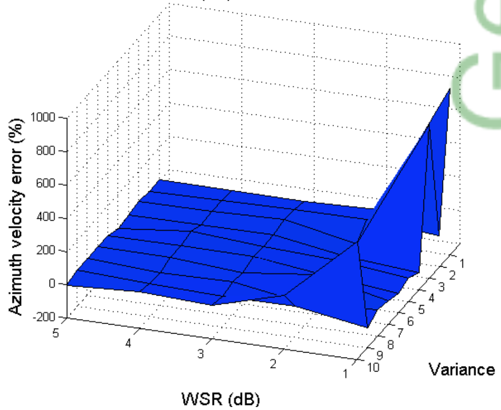
A sea scene is simulated using the gamma distribution

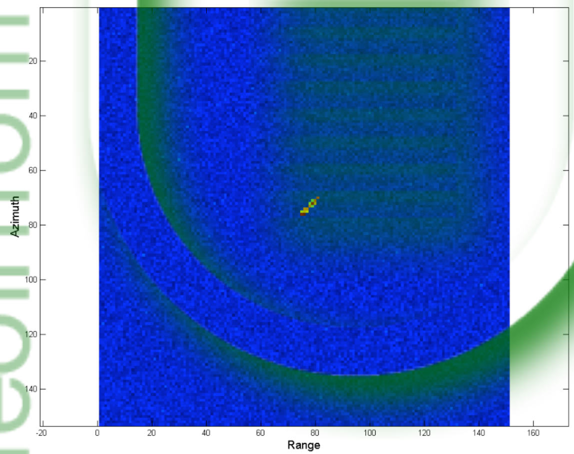
•To consider the speckle effect, we vary the variance of the gamma function

•To analyze the sensitivity to the wake visibility we vary the ratio between background mean level and the wake level (Wake-Sea Ratio, WSR).

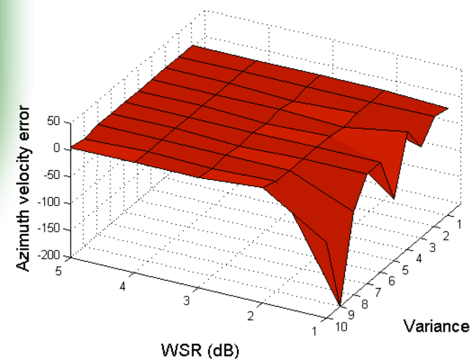
WOVE

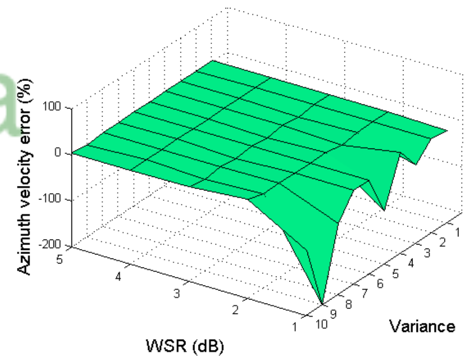






SOVE



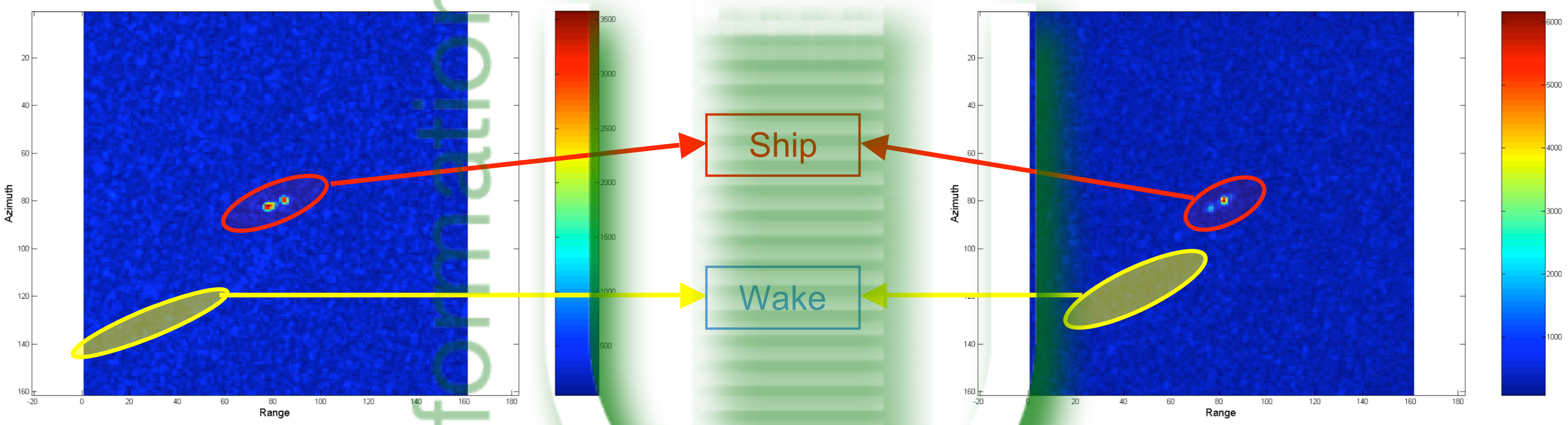


SOVE robust and reliable

WOVE accurate for WSR higher than -2 dB, SOVE gives better results

REAL DATA RESULTS

PRI ERS-2 frame 16466, orbit 2763



WOVE

Ship number	Lenght (m)	Real range velocity (m/s)	Real azimuth velocity (m/s)	Estimated range velocity (m/s)	Estimated azimuth velocity (m/s)	Range velocity error (%)	Azimuth velocity error (%)
1	163	6,7	-3,8	22,8	-3,6	239,1	-5,8
2	188	4,1	-2,7	-21,9	-12,1	-625,7	345,3

SOVE

Ship number	Lenght (m)	Real range velocity (m/s)	Real azimuth velocity (m/s)	Estimated range velocity (m/s)	Estimated azimuth velocity (m/s)	Range velocity error (%)	Azimuth velocity error (%)
1	163	6,7	-3,8	9,9	-3,4	47,8	-10,8
2	188	4,1	-2,7	3,8	-2,2	-7,1	-19,6

Note that the WSR is very low, the ships have a complex form and the velocity is small; therefore the velocity estimation results difficult for the algorithms, because the Radon transform can't identify well the wake as linear structure.

CONCLUSIONS

1) We presented two methodologies to estimate the velocity vector from raw data and amplitude data
2) The first algorithm was validated with simulated data: the analysis demonstrated that is not applicable to single channel. The algorithm presents a very strong motivation: the coupling between range and azimuth velocity must be considered
3) The second algorithm was validated with simulated and real data, producing very promising results
4) Future developments:

   <u>1° algorithm</u>: improve the selection criterion for the choice of the right filter, using also the phase information; develop an algorithm to retrieve the range velocity from the peak position in sub-aperture images.

   <u>2° algorithm</u>: improve the pre-processing, using dedicated filters and wavelet transforms to reduce the noise

PUBLICATIONS

[1] A. Radius, D. Solimini, “A velocity vector estimation algorithm tested on simulated SAR raw data”, Proceedings of the IEEE Int. Geoscience Remote Sensing Symposium, IGARSS, 23-27 July 2007.

[2] A.Radius, D.Solimini, P.A.C.Marques, “Radial Velocity Estimation Limitations from SAR subaperture”, IEEE Aerospace and Electronic Systems Transaction (submitted), 2008

[3] A.Radius, P.A.C.Marques, “A Novel Methodology for Full Velocity Vector Estimation of Ships Using SAR Data”, Proceedings of the 7th European Conference on Synthetic Aperture Radar, EUSAR’04 (accepted), 2-5 June 2008.

[4] A.Radius, P.A.C.Marques, “The SOVE algorithm for Full Velocity Vector Estimation of Ships Using Amplitude SAR Data”, Quartas Jornadas de Engenharia de Electrónica e Telecomunicações e de Computadores, JETC ’08 (submitted), 20-21 November 2008

MATHEMATICAL ANALYSIS

SAR PROCESSING BASIC THEORY

The target moves with radial velocity $v_{sr} = v_{rg} \cdot sen(\vartheta)$

Range migration: $R_{mov}(s) = R_c - \lambda \frac{(f_{Dc} - v_{sr})}{2}(s - s_c) - \lambda \frac{f_R}{2}\frac{(s - s_c)^2}{2}$

Range compressed data: $\hat{g}(s,t) = B \exp\left(-j4\pi \frac{R_{mov}(s)}{\lambda}\right) \cdot \mathrm{sinc}\left\{\pi B \left[t\text{-}2\frac{R_{mov}(s)}{\mathrm{c}}\right]\right\}$

Range migration compensation: selection of range time *t* to compensate the phase modulation induced from slant range time dependence

$$t(s,R_c^{'}) = \frac{2R(s)}{c} = \frac{2}{c}\left(R_c^{'} - \lambda \frac{f_{Dc}}{2}(s - s_c) - \lambda \frac{f_R}{2}\frac{(s - s_c)^2}{2}\right)$$

For $t = t(s, R_c^{'} = R_c)$ the dependence on the range variable is removed

$$\hat{g}(s) = B \exp\left(-j\frac{4\pi R_c}{\lambda}\right) \exp\left\{j2\pi\left[(f_{Dc} - v_{sr})(s - s_c) + f_R \frac{(s - s_c)^2}{2}\right]\right\} \mathrm{sinc}\left\{\frac{2\pi B}{c}\left[-\frac{\lambda v_{sr}}{2}(s - s_c)\right]\right\}$$

$s_{ci} = s_c \mp \frac{S}{2}(1 - \eta_{ap})$ $\quad S_i = \eta_{ap} S$

Look filtering:

$$\hat{g}_{filter}(s,t) = q \exp\left(-j\frac{4\pi R_c}{\lambda}\right) \exp\left\{j2\pi\left[(f_{Dc} - v_{sr})(s - s_{ci}) + f_R \frac{(s - s_{ci})^2}{2}\right]\right\} \mathrm{sinc}\left\{\pi B\left[t\text{-}\frac{2}{\mathrm{c}}\left(R_c - \lambda \frac{(f_{Dc} - v_{sr})}{2}(s - s_{ci}) - \lambda \frac{f_R}{2}\frac{(s - s_{ci})^2}{2}\right)\right]\right\}$$

$|s - s_{ci}| \le \frac{S_i}{2}$

i=1,2

After the range migration compensation we operate azimuth compression:

$$\zeta(s_c{'}|s_{ci}, R_c) = \int_{s_{ci} - S_i/2}^{s_{ci} + S_i/2} \hat{g}(s\,|\,s_{ci}, R_c)\, h_{az}^{-1}(s - s_{ci}{'}\,|\,s_{ci}, R_c)\, ds$$

$$h_{az}^{-1}\left(s - s_c^{'}\middle|s_c, R_c\right) = \exp\left\{-j2\pi\left[f_{Dc}(s - s_c^{'}) + f_R \frac{(s - s_c^{'})^2}{2}\right]\right\}$$

Two errors are made, choosing a stationary matched filter and compensating the range migration as for stationary target

$$\zeta(s_c{'}|s_{ci}, R_c) = q \exp\left(-j\frac{4\pi R_c}{\lambda}\right) \int_{s_{ci} - S_i/2}^{s_{ci} + S_i/2} \exp\left\{j2\pi\left[(f_{Dc} - v_{sr})(s - s_{ci}) + f_R \frac{(s - s_{ci})^2}{2}\right]\right\} \mathrm{sinc}\left\{\frac{2\pi B}{c}\left[-\frac{\lambda v_{sr}}{2}(s - s_{ci})\right]\right\} \exp\left\{-j2\pi\left[f_{Dc}(s - s_c{'}) + f_R \frac{(s - s_c{'})^2}{2}\right]\right\} ds$$

Compressed signal amplitude: [illegible]

Compressed signal phase: [illegible]