Notazioni complesse

campo em sinusoidale con pulsazione ω (sorgenti sinusoidali):

$$\mathbf{E}(t) = E_x(t)\mathbf{x}_0 + E_y(t)\mathbf{y}_0 + E_z(t)\mathbf{z}_0$$

$$E_x(t) = E_{0x}\cos(\omega t + \phi_x) \quad \ldots$$

$$\widehat{E}_x = E_{0x}\, e^{j\phi_x} = E_{0x}\,(\cos\phi_x + j\sin\phi_x) = E_{xr} + j\,E_{xj}$$

$$\widehat{E}_x\, e^{j\omega t} = E_{0x}\, e^{j(\omega t + \phi_x)} = E_{0x}\,[\cos(\omega t + \phi_x) + j\sin(\omega t + \phi_x)]$$

$$E_x(t) = \mathfrak{Re}\left[\widehat{E}_x\, e^{j\omega t}\right] \quad \ldots$$

$$\begin{aligned}
\widehat{\mathbf{E}} &= \widehat{E}_x\,\mathbf{x}_0 + \widehat{E}_y\,\mathbf{y}_0 + \widehat{E}_z\,\mathbf{z}_0 \\
&= (E_{xr} + j\,E_{xj})\,\mathbf{x}_0 + (E_{yr} + j\,E_{yj})\,\mathbf{y}_0 + (E_{zr} + j\,E_{zj})\,\mathbf{z}_0 \\
&= E_{xr}\,\mathbf{x}_0 + E_{yr}\,\mathbf{y}_0 + E_{zr}\,\mathbf{z}_0 \\
&\qquad + j\,(E_{xj}\,\mathbf{x}_0 + E_{yj}\,\mathbf{y}_0 + E_{zj}\,\mathbf{z}_0) \\
&= \mathbf{E}_r + j\,\mathbf{E}_j
\end{aligned}$$

$$\begin{aligned}
\mathbf{E}_r &= E_{0x}\cos\phi_x\,\mathbf{x}_0 + E_{0y}\cos\phi_y\,\mathbf{y}_0 + E_{0z}\cos\phi_z\,\mathbf{z}_0 \\
\mathbf{E}_j &= E_{0x}\sin\phi_x\,\mathbf{x}_0 + E_{0y}\sin\phi_y\,\mathbf{y}_0 + E_{0z}\sin\phi_z\,\mathbf{z}_0
\end{aligned}$$

$\hat{\mathbf{E}}$: quantità complessa di carattere vettoriale (*"vettore" complesso*)

"vettore" con componenti complesse,

non rappresentabile nello spazio ordinario

i vettori:

$$\mathbf{E}_r = E_{xr}\,\mathbf{x}_0 + E_{yr}\,\mathbf{y}_0 + E_{zr}\,\mathbf{z}_0 \quad \text{reale}$$
$$\mathbf{E}_j = E_{xj}\,\mathbf{x}_0 + E_{yj}\,\mathbf{y}_0 + E_{zj}\,\mathbf{z}_0 \quad \text{immaginario}$$

sono rappresentabili

vettore funzione del tempo:

$$\mathbf{E}(t) = \mathfrak{Re}\left[\hat{\mathbf{E}}\,e^{j\omega t}\right] = \mathfrak{Re}\left[(\mathbf{E}_r + j\,\mathbf{E}_j)\,(\cos\omega t + j\sin\omega t)\right] = \mathbf{E}_r\cos\omega t - \mathbf{E}_j\sin\omega t$$

$\mathbf{E}_r$, $\mathbf{E}_j$: si compongono a formare $\mathbf{E}(t)$ previa moltiplicazione per funzioni circolari di ωt
(non sono componenti di vettore euclideo)

ampiezza vettore in generale funzione del tempo

$$
\begin{aligned}
\mathbf{E}(t) &= \mathfrak{Re}\left[\hat{\mathbf{E}}\, e^{j\omega t}\right] = \mathfrak{Re}\left[(\mathbf{E}_\mathsf{r} + j\,\mathbf{E}_\mathsf{j})\,(\cos\omega t + j\sin\omega t)\right] \\
&= \mathbf{E}_\mathsf{r}\cos\omega t - \mathbf{E}_\mathsf{j}\sin\omega t \\
&= (E_{0x}\cos\phi_x\,\mathbf{x}_0 + E_{0y}\cos\phi_y\,\mathbf{y}_0 + E_{0z}\cos\phi_z\,\mathbf{z}_0)\cos\omega t \\
&\quad -(E_{0x}\sin\phi_x\,\mathbf{x}_0 + E_{0y}\sin\phi_y\,\mathbf{y}_0 + E_{0z}\sin\phi_z\,\mathbf{z}_0)\sin\omega t \\
&= E_{0x}(\cos\omega t\cos\phi_x - \sin\omega t\sin\phi_x)\,\mathbf{x}_0 + \\
&\quad E_{0y}(\cos\omega t\cos\phi_y - \sin\omega t\sin\phi_y)\,\mathbf{y}_0 + \\
&\quad E_{0z}(\cos\omega t\cos\phi_z - \sin\omega t\sin\phi_z)\,\mathbf{z}_0 \\
&= E_{0x}\cos(\omega t + \phi_x)\,\mathbf{x}_0 + E_{0y}\cos(\omega t + \phi_y)\,\mathbf{y}_0 + E_{0z}\cos(\omega t + \phi_z)\,\mathbf{z}_0
\end{aligned}
$$